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Between the clicks

Student learning paths when interacting with an adaptive learning resource in 4th grade mathematics

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Abstract

Adaptive learning technology has the potential to tailor learning to suit individual students' needs, desires and competence level. However, qualitative close up studies of students' interaction with adaptive technology are rare. This study explores 4th grade mathematics students' use of adaptive learning technology through screen recordings supplemented with analysis of data generated and tagged by the adaptive engine. The study explores how different types of students, i.e. students with varying mathematical competence levels and motivation towards mathematics, interact with an adaptive learning material, what learning paths emerge for different types of students in their interaction with the learning resource and how student self-efficacy is affected by the interaction. The study shows that students neglect to access and utilize the supportive resources in the learning tool. Rhapsode worked poorly for a student with a combination of low mathematical competence and motivation for mathematics but fairly well for the other three students.

Adaptiv læringsteknologi har potentiale til at skræddersy læring, så den passer til den enkelte elevs behov, ønsker og kompetenceniveau. Kvalitative nærstudier af elevers interaktion med adaptiv teknologi er dog sjældne. Denne undersøgelse udforsker 4. klasses matematikelevers brug af adaptiv læringsteknologi gennem skærmoptagelser suppleret med analyse af data genereret og tagget af den adaptive motor. Undersøgelsen undersøger, hvordan forskellige typer af elever, det vil sige elever med varierende matematiske kompetenceniveauer og motivation i forhold til matematikfaget, interagerer med et adaptivt læremiddel, hvilke læringsveje der opstår for forskellige typer af elever i deres interaktion med den adaptive læringsressource og hvordan elevens self-efficacy påvirkes af interaktionen. Implikationer for både fremtidig design og læreres brug af adaptive læremidler diskuteres.

Between the clicks

Student learning paths when interacting with an adaptive learning resource in 4th grade mathematics

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Adaptive learning, pedagogy and student interaction

A defining characteristic of adaptive learning technology is the dynamic change in system response based on user interaction (Liu et al., 2017). This feature of adaptive learning technology has the potential to personalize learning environments and differentiate challenges and support which, in turn, can lead to strengthening of student competence, motivation and confidence (Pollard & James, 2004; U.S. Department of Education, 2017). The idea that content, learning context, pedagogical approach etc. could be tailored to suit every students' needs and interests holds great appeal and promise.

Paradoxically, research in adaptive technology rarely focuses on observations of individual students, but rather on test data and survey responses (Martin et al., 2020). If we want to understand which groups of students can benefit from using adaptive learning technology and what their interaction can tell us in relation to designing and optimizing adaptive learning designs, a reasonable approach would be to take a closer look at what students actually do when interacting with an adaptive technology.

The overall effects of using ICT in school are ambiguous (Bulman & Fairlie, 2016; OECD, 2015). However, Tamim et al (2011) found multiple studies showing positive effects of ICT-usage and concluded that results depend on contextual factors such as pedagogy, content, and teacher competence. Similarly, Gericke al (2014) concluded that positive and negative effects depend on teaching method and context. One consequence of these findings could be that we need to study use of digital technology for specific educational purposes in rather specific contexts instead of expecting ICT in general to have a positive impact.

However, not all educational technologies are designed to involve teachers in professional judgement and adaptation, and some products may even be designed to offset the role of contextual factors such as heterogeneous student proficiency levels and teacher beliefs. Adaptive technologies are examples of technologies that potentially make teacher intervention superfluous vis-à-vis use of the learning resource (Apoki et al., 2022). Adaptive learning technologies are designed to tailor the learning experience to individual students' learning needs by adapting learning paths based on tracking of students' interaction and input (Somyürek, 2015). Adaptive technologies aspire towards optimal personalized learning experiences by providing immediate and relevant assistance, resources and feedback (Kerr, 2016; Walkington, 2013). Hence, adaptive technologies have the potential to act as a digital *tutor* (Taylor, 1980), i.e. take care of core didactic functions, such as selecting content, presenting content in an optimal fashion, evaluation, differentiation etc.

Research in adaptive learning technologies is typically preoccupied with optimizing the technology to maximize learning gains for the users (e.g. Conejo et al., 2004; Guzmán et al, 2007; Liu et al., 2017; Peng et al., 2019; Tai et al., 2001). Studies about use and outcome of adaptive learning resources in elementary school are limited both in number and robustness (Holmes et al., 2018). Much research in the field is explorative and small scale (Verdu et al., 2008). The vast majority of studies are with students in higher education (Johnson & Samora, 2016; Xie, et al., 2019). It is not surprising, that trials have mainly been conducted with students on higher educational levels, as we could expect the outcome of prolonged individual interaction with a digital adaptive learning resource to depend on levels of self-discipline and meta-cognitive abilities that few primary schools students can fulfill. Overall, these studies indicate positive learning outcomes, but highlight the importance of the facilitation and instruction of educators (Kulik & Fletcher, 2016; Verdu et al., 2008, Wang et al., 2020).

Du Boulay (2019) distinguishes between screen level pedagogy and classroom level pedagogy in relation to adaptive learning. Screen level pedagogy encompasses the interaction between the individual student and the adaptive learning resource which, in turn, depends on the pedagogical approach embedded in the learning resource. Classroom pedagogy is the teacher's realm; the teachers connect and integrate the activities and content from the adaptive learning resource in their broader educational design. The teacher, however, is to some degree left out of the screen level learning situation because the adaptivity depends on interaction between the individual student and the adaptive learning resource.

A recent study shows, that teachers experience that the individualization connected with using adaptive learning resources challenges established classroom norms of collectivity, and that teachers feel a lack of control and insight vis-à-vis their students' learning process (Modén, 2021). Therefore, it would seem relevant to study the interaction between the individual student and the adaptive learning resource to gain insights into what goes on when a student interacts with an adaptive learning resource and to focus research on the student as the key to positive outcomes of using adaptive technologies.

In a literature review, Nakic et al. (2015) investigated the various variables and student characteristics that adaptive technologies should employ to create valid *user models* (also known as learner models), i.e. a set of assumptions about the user inferred from user interaction with the learning resource that guide what is presented for the user. It was found that cognitive ability and personality are the user characteristics that attract attention from most researchers, but that research in the last two decades is increasingly preoccupied with noncognitive characteristics such as emotional, motivational and meta-cognitive factors, in user models.

Learner models are typically constructed based on a combination of system identification of relatively stable user characteristics such as learning style and more variable factors such as learning history and student knowledge (Nguyen, 2015). Data generated by user interaction with the adaptive learning resource and other data need to be interpreted and, in turn, translated into system action. However, the data points used by the system to create and adjust the learner model will in themselves be selected proxies of relevant variables. For example, average user time spent on task items can provide indications of user domain specific ability, motivation, self-efficacy etc. System interpretation of singular user actions is uncertain. Prolonged time spent on a task can be caused by the student's thoroughness, uncertainty, inactivity or distraction, that the student finds the item challenging etc. In essence, the learner model will be a rudimentary image of the student, although perhaps sufficient for the adaptive purposes at hand.

In this study, we compare the data provided by the adaptive learning resource, Rhapsode, to close observations of students interacting with the adaptive learning engine. The aim is to explore what characterizes different types of students' behavior when using the learning resource, by studying what happens between the clicks and other user input that the machine uses for learning analytics, and how the learning resource responds to student actions. This close up investigation of student behavior using an adaptive learning material seems particularly relevant in relation to a technology that intends to personalize learning based on students' characteristics and behavior. Furthermore, the study aims to investigate how students' self-efficacy and motivation is affected by using the adaptive learning resource. Self-efficacy is the perceived capability of an individual to perform given actions (Schunk, 1991). Self-efficacy is interwoven with both academic motivation and academic proficiency. In the seminal theory of Bandura (1977), the individual's expectation of personal efficacy determines if the individual will exhibit coping behavior and invest the necessary and prolonged effort to overcome challenges and adversity. Self-efficacy can be strengthened when a person experiences ability to overcome challenges and coping with threatening situations, which, in turn, can improve academic proficiency, because the behaviors associated with self-efficacy are fundamental to benefiting from learning situations.

Viewing the adaptive learning technology *Rhapsode* through the lens of self-efficacy theory will serve to point out strengths and weaknesses in the learning material towards keeping students' self-efficacy intact or even strengthened and, in turn, keep students motivated and able to learn. Bandura (1977) identifies four sources of self-efficacy: performance accomplishments, vicarious experience, verbal persuasion, and physiological states, with the personal experience of success in performance being the strongest source of self-efficacy.

Hence, the focus of the study is on how students with different combinations of mathematical competence and motivation cope with Rhapsode's way of organizing a learning process and, conversely, how the learning tool copes with students with different prerequisites. However, the mathematics subject is a backdrop for studying student behavior and interaction with an adaptive learning technology. Therefore, we do not refer to previous studies of digitally mediated learning in the mathematics subject, mathematical self-efficacy etc.

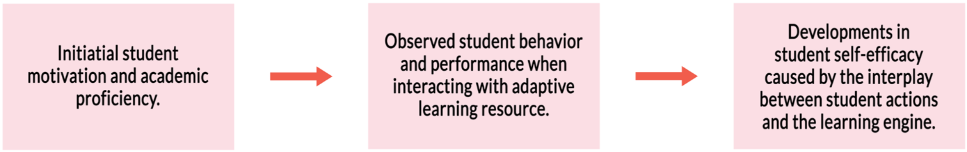
The research question of the study is as follows

How do different types of students interact with an adaptive learning material, what learning paths emerge for different types of students in their interaction with the adaptive learning resource and how is student self-efficacy affected by the interaction?

Method

In this study, we explore how Danish grade 4 students perform and interact with the digital adaptive learning resource, *Rhapsode*, for mathematics. We aim to understand how four students use the adaptive learning resource, more specifically how they utilize the resources made available by the learning material and go about solving the tasks. Furthermore, we aim to understand how students of varying mathematical proficiency and motivation towards the mathematics subject interact with the adaptive learning resource and how student self-efficacy is affected by student interaction with the adaptive learning engine (Figure 1). *Student variables* in this study are baseline and situational academic motivation towards mathematics and academic proficiency towards mathematics. The relevant *situation variables* in the learning situation are student interaction with available resources, i.e. tasks and resources providing explanations, assistance/modeling and feedback, which are delivered by the adaptive learning engine.

Figure 1.
Components in the study.



Data

Because Rhapsode is an adaptive learning resource, potentially students will be supported and challenged appropriately based on their performance and input, as soon as the adaptive engine has determined student proficiency level. Rhapsode monitors student level and progress in knowledge (what the student knows and understands), grit (the ability to persist despite difficulties), as well as meta-learning (students' ability to assess what they understand and are capable of). The teaching aid collects data in relation to the connection between the student's perceived level of certainty on whether they know or can perform specific knowledge bits or tasks and the learning resource offers an explicit, immediate assessment of student performance. How the data is interpreted by the learning resource and transformed into action, we do not know.

The results of the auto generated analysis of student progress and performance is made available in the teachers' dashboard, but in an aggregated and processed format that does not allow to gain insights into individual paths in the learning resource. However, we did gain access from the developer, Area9, to datasheets showing tagging of learning objects and the raw data generated by Rhapsode. Each learning object is tagged with a specific learning objective. Furthermore, each learning object is tagged in a content category, i.e. which type of learning object (explanation, multiple-choice question, tasks that require multiple answers etc.).

The adaptive engine generates data on:
— The exact time the student enters a learning object and time total spent on the learning object.
— Student score (0-100%) and result (correct, wrong or partially correctness in %) in relation to each learning object.
— Student self-assessment of self-confidence in relation to their understanding of explanations and their degree of certainty that they answered a task correctly.

Figure 2.
Rhapsode interactive explanation of parallelog-ram.

Rhapsode uses user responses and clicks to generate data about the abovementioned elements. However, much user interaction and activity is not tracked in the activity log. Firstly, there are numerous interaction possibilities on each page, which are not recorded by the learning material. Figure 2 shows a screen dump of an explanation of the parallelogram. In the left side of the screen, there is a verbal explanation. The student can choose to have it read aloud and set it to read aloud automatically when entering the page. The student can scroll down and read the written representation in the box. In the center screen the student can check the box below the figure and see the extended lines (blue), which illustrates that the sides are parallel. Furthermore, the explanation consists of three slides, each with similar verbal explanations and illustrations. The student has to click to proceed to the next slide. Nevertheless, the learning material activity log will only record total time spent and student self-assessment of understanding at the end of the slide show. Furthermore, student understanding of the explanation is not tested in conjunction with the explanation object, so there is no appraisal for student effort.

We use screen recordings of student interaction (n=4) with the learning material to observe student behavior. This allows a qualitative task-by-task investigation of student learning trajectories in the digital milieu. The task-by-task analysis is supplemented with a continuous monitoring of students' self-reported self-efficacy level.

Before submitting an answer to each task and after each presented explanation, students must assess their self-efficacy by evaluating how sure they are, that they solved the task correctly or how sure they are, that they understand the explanation put forward by the learning resource.

Screen recording is an unobtrusive method of generating data on user interaction with a digital learning resource. Compared to using for example eye tracking technology to determine what students are fixating upon and for how long, screen recording allows students to work unattended in their natural environment and they do not need to physically wear research equipment. Gaze and mouse movement has been found to interact sufficiently to provide similar information as eye-tracking as a proxy for attention in natural tracing tasks, i.e. when subjects are not instructed to consciously move their eyes (Demšar & Çöltekin, 2017). Eye and mouse movements have been found to be highly coordinated with close to 0 pixel gaze/cursor distance in online search tasks (Rodden et al. 2012). Still, analyzing screen recordings to determine student attention will only provide a rough understanding of student attention. The present study does not claim accuracy in number of seconds spent attending to given objects but rather to present a comprehensive impression of students' behavior when using the learning resource.

An adaptive learning environment such as *Rhapsode* provides the opportunity to study student self-efficacy and motivation through their behavior and performance in the *Rhapsode* learning environment:

— We can observe students' initial coping with mathematical challenges.
— We can track students' development in self-efficacy and motivation as they are making their way through the learning modules based on their actions and self-reported evaluation of their degree of certainty, that they have understood the content or solved the task correctly, which also gives us an indication of students' perceived self-efficacy.

For both points of interest, the interplay between student and learning resource must be taken into account, as the learning resource reacts to student actions. In analyzing the recordings, we first made a qualitative description of students' actions and events in relation to each exposure to a learning object. We also used all the above mentioned data on student performance and action generated by Rhapsode. To contextualize our understanding of student behavior and self-efficacy we use a baseline test and survey measuring student proficiency level in mathematics and academic motivation towards the mathematics subject.

Procedure

We administered a test of mathematics skills using a standardized test battery, *Matematikprofilen [Mathematics profile]* (Gyldendal, n.d.a, n.d.b, n.d.c) to all students in the 4th grade class. In the test, the students' skills, knowledge and competences are measured in relation to the Danish National Mathematics curriculum within the mathematical subject areas: 1) numbers and algebra, 2) geometry and measurement, and 3) statistics and probability. The test consists of both closed tasks (multiple choice) and open tasks where the student must draw and/or write the correct answer. The closed tasks consist of dichotomous items (i.e. true/false), while the open tasks consist of polytomous items, where a number of points are given (e.g. 3-2-1-0) based on examples of what characterizes a task solution on different levels. The test gives a total point score that can be used to divide the students into one of five levels of subject mastery. Students had three hours to complete the test, which was administered by their regular teacher.

To measure the students' self-perceived competence in mathematics, we used a scale consisting of nine items from the international survey Trends in International Mathematics and Science Study (TIMSS) (Martin et al., 2016). The scale is used in TIMSS to compare students' self-perceived competence in mathematics across countries for both 4th and 8th grade students. In the questionnaire, the students were asked to decide on all nine statements, which in different ways are about the students' assessment of their own ability (e.g. "I usually am good at mathematics", "Mathematics is more difficult for me than it is for many of my comrades"). To measure the students' motivation towards mathematics, we used a scale consisting of nine items from TIMSS (e.g. "I like learning mathematics", "I wish I didn't have to learn mathematics") (Martin et al., 2016).

The four students screen recorded were selected at random based on a stratification that divides the students according to two parameters:

1. Whether the students' baseline math test showed a relatively high or low level of math competence.
2. Whether the students' responses to our baseline survey showed relatively high or low motivation in relation to the mathematics subject.

This gives four possible combinations of competence level and motivation (see Table 1), and one student has been selected from each of the four fields.

Table 1.
4th grade students selected for screen recording.

		Competence level		
		Low	Average	High
	1/3 lowest	Student 1		Student 3
Motivation	1/3 average			
	1/3 highest	Student 2		Student 4

Note: The students were selected by simple random selection in each of the four strata.

Hence, the strategy for case selection is maximum variation (Flyvbjerg, 2006) in an attempt to shed light on how student interaction differs with different student characteristics regarding competence level and motivation – and how the learning resource adapts to different types of students.

The study was carried out during the Covid-19 pandemic, more specifically when Danish students were attending school from home. Therefore, students' interaction with Rhapsode was recorded in the students' home, which means that they were not monitored by either researcher or teacher. Therefore, students had no possibility of cooperation and 'cheating' the computer through help from peers, a strategy which has been observed in a strain of the project the present study is part of (Gissel et al., 2020). The students freely choose a learning module for the recording, which they themselves carried out following our instructions. Rhapsode is primarily used for learner self-study, which means that the setup is similar to the usage situation intended by the producer of the learning resource. Students were instructed to use the learning resource for a minimum of 45 minutes, which all four students did.

Analysis

The learning material's sequencing of tasks and explanations
The analyses of the data generated by the screen recordings firstly explore students' learning paths across the four cases when using Rhapsode. As the algorithm behind Rhapsode is a company secret, this endeavor can be viewed as an inductive approach to try to understand how this particular algorithm works.

As a starting point we use the structure and taxonomy of learning modules. Each learning module has a rather narrow subject, e.g. fractions (parts 1-2) or coordinate system. The learning content is divided into multiple learning objectives (some 30 for each module) which in turn are associated with specific content, i.e. interactive questions, multiple choices tasks, math problems or explanations. As these partial learning objectives are structured according to a taxonomy, it is possible to track and understand the order in which Rhapsode presents this micro granulated content as well as which and when content is presented repeatedly.

To illustrate how the algorithm works student 1's interaction with the teaching aid will be analyzed to make general points about Rhapsode's way of presenting content. These points about Rhapsode's way of working thus apply to all four case students. In addition, this case description thoroughly explains which types of tasks student 1 has been exposed to in order to describe the content in the Rhapsode mathematics courses.

Student 1 chose to work with the "Division 2" module for the screen recording session. During the 45 minutes of recording, the student is involved in 59 activities (in two different modules). In Rhapsode activities are grouped into different types which are described in the following along with student 1's exposure to each type:

— 13 of the 59 activities are communicative texts where the student's knowledge is not tested (but the student must evaluate his own understanding).
— 10 are tasks that require a single answer.
— 4 are multiple-choice tasks that check student understanding.
— 11 are assignments that require the student to write two or more answers (e.g. partial calculations of a division piece that is regrouped).
— 8 are sequences where the student is scaffolded by carrying out step-by-step tasks towards a unified, more complex whole, typically a mathematical procedure.

— 13 are so-called *math problems*, which require a little more student independence to be solved (e.g., the student must write an arithmetical problem which has then has to be calculated).

In the Module "Division 2", the student has worked with a mathematical subject area which, from Area9's side, is divided into 25 learning objectives. In Appendix 1 it is explained and shown how the analytical findings are represented. Appendix 1, Table 1 shows that Rhapsode's way of putting together a learning process is atypical compared to digital skill-and-drill learning materials, in which a specific content area and a degree of difficulty is chosen, and then the student solves training tasks of increasing difficulty within the area (Gissel & Skovmand, 2018). In Rhapsode, a larger subject area is, as mentioned, granulated into smaller learning objectives. Furthermore, in Rhapsode's dashboard, you can see that the learning objectives of a given area are arranged in a taxonomy from the simple to the more complex.

However, the students' progress through the learning objectives in Rhapsode is by no means straightforward and linear, as in a typical training learning tool where students work with the same procedures with increasing difficulty level (Gissel & Skovmand, 2018). Rhapsode represents the content area as a network structure, where the student is thrown around between learning goals (points in the network) and up and down in taxonomic level and degree of difficulty.

Thus, we can see that the first task encountered by student 1 relates to learning objective 2.2. Rhapsode tries to find the student's level and the first probing is not in relation to the easiest learning goal, but rather on the top edge of the lowest third. Student 1 answers the first task incorrectly, but the teaching aid does not jump down to the lowest targets for that reason either. Instead, the student gets an explanation in relation to learning objective 2.2. and is then presented with a task in connection with objective 2.3, which the student answers correctly.

Typically, in a session, at the start there is an overrepresentation of tasks from the lower taxonomic levels with detours around the area. Likewise for student 1. If the student answers a few tasks incorrectly, the student typically gets an explanation and then more tasks. However, assignments and explanations never follow one another directly. Typically, for example, it takes 5-6 activities before the student gets a task that tests him in the learning target content that has been explained. And the other way around; in case of an incorrect answer within a learning objective, the student does 5-6 activities in relation to other learning objectives than the one he answered incorrectly within, and then there is either an explanation or a repetition of the mathematical problem (possibly with other numeric values).

In addition, the student is quickly pushed on to new learning goals by Rhapsode. The teaching aid stops challenging the student within a mathematical sub-area as soon as the teaching aid registers or believes that the student has mastered it. The teaching aid will typically consider a learning objective as completed after a single correct answer to a task.

This can be seen, for example, in student 1's meeting with learning objectives from areas 1.1 and 1.2, which the student answers correctly in the first attempt. This is the only time the student meets these two learning objectives. He also gets a few explanations regarding goals in area 2, Division of whole 10s... and answers (apart from a single slip) correctly to tasks from area 2.1-2.3. He gets these done quite quickly (by the 17th activity these are finished). In general, it can also be seen that even if the student answers incorrectly to the first task within a learning goal, it only requires one correct answer from the student before the teaching aid sends the student on to new goals. Sometimes the student has two correct answers before he is sent on.

Another characteristic of the sequencing of activities is that, in the short term, the teaching aid does not let the student hang on to the same learning goal if the student answers incorrectly. In the short term, the student is pushed on - often to learning goals that are taxonomically above what the student has answered incorrectly. This can be seen, for example, in connection with the student's attempt at learning objective 1.3. What happens is that the student answers incorrectly to the first three tasks, but correctly to the fourth, and then the learning tool moves on to other objectives. The student gets his first task within 1.3 "Perform simple division with remainder" (The task is called: 10/3=? Remainder=?) as the 14th activity, the next one after the first wrong answer is presented to the student as the 25th element, and again as the 31st followed and then the last time with a similar interval where the student finally gets it right.

Although the learning path is thus by no means straight but rather unpredictable and must seem varied and perhaps random to the student, since the student cannot know what to expect from the next task, there is a large degree of continuity in the many repetitions of tasks, if the student answers incorrectly. For student 1, there is in relation to learning objective 3.3. five exposures to the same task; generally, students will face the same tasks that they answer incorrectly over and over again until they answer correctly.

Another example of how the learning paths in Rhapsode take shape can be seen in student 1s exposure to learning objective 1.4, which is about using division with remainder in everyday situations. Here, the student first encounters an explanation where the student's knowledge is not tested. In a linear design, such an explanation would be followed by tasks where the student must apply what has been explained or show that it is understood. However, in Rhapsode, the student does not initially get a task where the student has to apply what he has learned in the video. Rather, the student is exposed to three tasks and one explanation related to learning objectives 3.1-3.5, which should be more difficult than objective 1.4. When learning objective 1.4. is revisited the student answers only partially correct. The student then is presented with five tasks from other clusters of objectives before encountering 1.4. again.

Individual student learning paths using Rhapsode

Because Rhapsode is an adaptive learning tool and because case selection aims for maximum variation, we would expect the learning paths to differ depending on student performance and characteristics.

Student 1's learning path and development in self-efficacy
Student 1 placed in the lowest third of the class both in terms of mathematical competence and motivation for the mathematics subject. At the start of the session (the first 14 activities), the student works quite thoroughly and has a high success rate in relation to answering the tasks. He reads the assignment texts, thinks carefully when he is in doubt, and on the fairly simple passages at the beginning of the course, he goes straight to the answer. In other words, the student gets off to a good start.

However, around activity 15, which is a step-by-step, scaffolded task that shows, using graphic representations, how to divide by dividing pieces by 10s, the student is challenged. In the first part of the task, the student revises his answer several times, and ends up writing an incorrect result. In the self-assessment, the student marks "I do not have a clue". Some parts of the task the student gets right, others wrong. When he answers incorrectly, he alternates between ignoring the feedback and examining it quite thoroughly. But in the last part of the task, he spends only a short time on feedback and neither accesses the resource "See more here" nor reads the explanation on the left side of the screen.

From here, the student's progress is characterized by a lack of concentration and thoroughness in reading instructions, explanations and examining feedback. It is a vicious circle, as he approaches the tasks in a skewed way the first time he sees them, and does not become much wiser through his attempts, whereby there is a basis for a series of defeats in relation to the same task. For the multiple-choice question in activity no. 25, for example, he randomly clicks around between the options, at some point sends the answer and does not consult the subsequent feedback. Or when the explanation for learning objective 3.1 is repeated after the student has answered a few tasks incorrectly, he does make the effort to understand the explanation, but quickly clicks his way through all the slides and marks “I know” in the self-assessment. A little later, the teaching aid repeats the explanation, perhaps because the student only spent 6 seconds on the last encounter with the explanation. Normally, an explanation would be followed by a test item. Nevertheless, the student does the same when the explanation is repeated, quickly clicks through it and marks “Got it”. When the student is next given a task related to the learning objective, the student must solve a math problem that involves starting by making four piles with 10 in each, and he must decide how many he has distributed. The student hesitates for a while and then writes a lot of numbers and letters in the writing field and deletes it again. Clicks around the page occasionally. Finally writes “40”, which is correct, and ticks “I think I know”.

Overall, it seems as if the student’s self-confidence is declining; his self-assessments are lower and lower as the course progresses: he marks “Not sure”, “I think I know” or “I do not have a clue”. Even if the teaching aid prompts him to try to revise his answer based on a hint in the feedback section, the student quickly clicks on. Occasionally he invests energy in answering correctly, but rarely succeeds. On several occasions, he clicks around on the screen’s action points instead of being on task. This is where the student’s recording ends. The student’s motivation thus quickly seems to fade in the event of adversity, and he does not seem motivated to complete the tasks, as he rarely invests the necessary effort.

Another characteristic feature of this student’s progress in the teaching aid is that the student does not prioritize familiarizing himself with the explanations that are available. The student typically doesn’t spend time looking at feedback or revising his wrong answers, nor does he draw on the resources that are available. The student does not seem to expect or be interested in learning anything he does not already know, and this means that the student is actually practicing very basic mathematical operations (e.g. dividing a simple piece) without understanding what he will use them for, for example, that he can use regrouping to calculate more difficult calculations.

As can be seen in Table 2, the student does not finish the module in the 45 minutes. Several of the learning objectives are still active due to wrong answers and five of the learning objectives have not been touched at all yet when the recording ends. Therefore, there is still a long way to go for student 1. Nevertheless, we can also see from some of the previous learning objectives that the student actually manages to answer correctly, for example with learning objectives 3.1 and 3.2. Here it should be noted that the student is exposed to several repetitions of the same tasks and has access to see the correct answers and often also a recommendation for procedure. Whether the student actually understands how to transfer what he is doing to other situation, or whether he can remember the result/procedure, remains uncertain.

In summary, the teaching aid does not seem to be able to maintain the motivation of the student with a relatively low level of mathematical competence and motivation for the mathematics subject. Furthermore, the student does not develop mastery during the session, but rather loses self-confidence in relation to his mathematical self-efficacy (Mozahem et al., 2021).

Student 2's learning path and development in self-efficacy

Student 2 scored in the lowest third on the mathematics test, but is high in self-reported motivation in relation to the mathematics subject. The student chose to work with the module "Multiplication 2". The student's characteristics help explain his behavior in Rhapsode. The student fairly consistently overestimates his own abilities, has fairly high self-confidence and a performance-oriented (but not mastery-oriented) motivation, but lacks basic math skills/understanding.

In the module, the student worked with a total of 23 learning objectives (see Appendix 2, Table 1). The student has a high cadence. He manages to be in contact with 114 learning objects in the 45 recorded minutes. In comparison, student 1 managed 59. He answers many tasks and is not afraid to try to answer, and to answer quickly – even without familiarizing himself with the task formulations. He also goes through the explanations very quickly, very often without familiarizing himself with them. The student thus shows great motivation in relation to answering the tasks. Also, student 2 marks a high degree of self-efficacy in the student's self-assessments in Rhapsode. Thus, the student quite consistently marks that he "knew it" in advance during explanations, and that he is sure that he answers the tasks correctly.

Student 2 gets off to a relatively good start. At the start of the session, the student takes plenty of time to read explanations and solve the tasks. The student also looks at part of the feedback for incorrect answers. This indicates motivation to learn. However, right from the start he gets a lot of incorrect answers when he is challenged beyond target area 1, Commutative law. By task 14, however, it seems that the student begins to doubt his own abilities. He is faced with a passage that is slightly higher in the learning target taxonomy than the previous tasks. The student fills in the wrong answer and for once marks "I think I know it".

Around activity 33-51, Rhapsode stays within learning objective areas 2-3. This is probably because the student answers the majority of the tasks incorrectly. After a quarter of an hour, the teaching aid begins to push the student up the taxonomic ladder anyway, even if the student's poor performance does not justify an increase in the degree of difficulty. This means that the student is challenged beyond his abilities in long passages. Around activity 61, the student also atypically marks "Not sure" in the self-evaluation and answers incorrectly. This could be a sign that he is losing his confidence, which was otherwise at its peak in the first half of the recording.

Student 2 is clearly challenged. He gets a total of 9 exposures for the same task, albeit with different numerical values: The tasks are variations on the piece 800x400, i.e. multiplication of whole 100s. In addition, the student receives the same explanation a total of three times. Nevertheless, the first eight times he encounters the task, he answers incorrectly.

The student thus makes an effort to answer, but not to a great extent to familiarize himself with the explanations and consult feedback. It's amazing how long he can maintain the belief that he can answer the same tasks correctly, despite repeated feedback to the contrary. Gradually the student begins to doubt himself. This can also be seen by the fact that he changes a correct to an incorrect answer twice before submitting his answer.

The student melts down at the end of the 45 minute session. He has been given many repetitions of the same tasks to which he has answered incorrectly. In the end, he apparently doesn't care anymore. In the process, however, he shows a striking persistence despite being constantly told that he is answering incorrectly. He has high belief in his own abilities until the last third of the recording. He overestimates himself. He does not look carefully at explanations and usually not at feedback either, but answers the next tasks without hesitation and is usually sure that he answers correctly. This student probably leaves the session with lower self-efficacy than when he started.

Student 3's learning path and development in self-efficacy

Student 3 is characterized by good math skills but low motivation for the math subject. The student chooses to work with the module "Coordinate system". However, the student manages to complete the module in around 18 minutes, which is why he also begins the module "Perimeter and area". In the analysis, we will concentrate on his progress through the module "Coordinate system" as it shows how the learning tool challenges the student, who has either already learned or can easily understand the content and answer the tasks. In the "Coordinate system" module, the student has worked with 25 learning objectives (see Appendix 3, Table 1).

The student completes the 25 learning objectives through only 45 interactions with learning objects. As can be seen in Appendix 3, he only gets a single exposure to learning objects in relation to 13 of the 25 learning objectives. The value of the learning tool not wasting the skilled student's time; the student is quickly sent on when the teaching aid finds that he has mastered an objective.

Given the observation that students are typically sent on quickly when they can answer a single task correctly within a learning objective, there is a rather surprising course around the learning objective "Plot a point given its coordinates" (4.3). The student gets off to a good start. He gets an explanation of how to plot points into a coordinate system. At the end of the explanation is a task that he answers correctly. 5 tasks later comes a task that tests whether he can apply what he was told in the explanation – he answers this correctly. He is then given an identical task (with the same values), which he also answers correctly. Right after, the explanation is repeated with the task for the same learning objective, where the student simply clicks on instead of solving the task (which is understandable given the history). Then the student is exposed to the same task as the previous two.

The student displays a high level of metacognition. He does not have many mistakes and is also self-confident in his assessment of his own achievements, as he marks "Know it" or "Knew it" a total of 39 times. Typically, he shows uncertainty about the tasks in which he makes mistakes. All in all, the student gets through the module fairly quickly. It seems like he knew the material well and just needed a refresher. Student 3 gets refreshed or quickly learned the few things that he has forgotten or could not do beforehand. As the student is not challenged largely by the module and as he can accurately assess his own competencies and uncertainties his self-efficacy would seem to be consolidated or heightened through the session.

Student 4's learning path and development in self-efficacy

According to the test result and survey, student 4 is characterized by a relatively high mathematical competence level and high motivation for the mathematics subject. The student chose to work with the module "Division 2" where she worked with 26 learning objectives. The student managed to be in contact with a total of 56 learning objects in the 45 minutes she recorded (Appendix 4, Table 1).

Student 4 is quite thorough in her approach to answering the tasks, as she thinks carefully before answering. She also spends quite a lot of time familiarizing herself with the various resources – including explanations. The student generally makes a great effort to answer correctly and understand. However, at an early stage, the student clicks away the explanation on the left side (Figure 2) and it remains gone for the rest of the session.

The student gets some wrong answers at the start of the session, and the teaching aid presents the student with quite a few explanations. The student also answers incorrectly to some of the tasks that test the student's understanding in connection with explanations. The student thus does not have an easy time. It is thus the student's rather high degree of self-efficacy that pulls her through, and her thoroughness could indicate that she is mastery-oriented.

The student exhibits a high degree of metacognition. She marks relevant and precise in the self-assessments of how certain she is that she has answered correctly or understood an explanation when she is not sure. Only three times does she mark "Know that" for an explanation, and a total of 34 times she assesses that she "Thinks it is right", what she has answered in an assignment. "Completely blank" is marked six times. This humility or caution may be the reason why Rhapsode, with this particular student, in several cases provides additional tasks within a learning objective, even if the student has managed to answer correctly once. If the student does not feel confident, the student is given tasks to consolidate her knowledge and skills. However, it cannot be seen that the student increases her self-efficacy or belief in herself through the session. But in this student's case, motivation and self-efficacy translate into thoroughness and care in relation to familiarizing herself with the resources, whereby the learning tool works well for the student, who slowly but surely works her way through the learning objectives.

Concluding discussion

The study has shown how assignments and explanations are presented to the student in Rhapsode. The learning tool probes the student from the start to determine competence level. Thus, the learning tool does not start with the easiest learning objectives. Moreover, the student is constantly sent around between different learning objectives, which can be both high and low in the producer's taxonomic arrangement of learning objectives.

The student rarely encounters tasks related to the same learning objectives in succession. In the case of an incorrect answer, the student is met with an explanation or a repetition of the task (most often with new numerical values), but these only come after the student has dealt with a variety of tasks and explanations related to other learning goals. Typically, approximately six activities will pass before the learning objective is taken up again.

Students are quickly sent on to new learning goals. A single correct answer is enough for a learning objective to be considered completed in Rhapsode – unless the student himself expresses doubts in his self-assessment. This may cause student failure to experience mastery; for some students, they have answered several tasks incorrectly, but when they finally get a correct answer within a learning objective, Rhapsode moves on to other objectives rather than allowing the student to consolidate. The need to consolidate and experience mastery should be considered in future designs.

In general, students neglect to access and utilize the supportive resources offered by the learning tool. Everyone ignores the explanatory box on the left side of the screen, the opportunity to revise answers after feedback is rarely used, and it is certainly not consistent that students examine the feedback they receive.

Since uniform tasks are repeated in the event of incorrect answers from the student, and potentially repeated many times, it can be difficult to know whether the student fundamentally understands the mathematical connections that the task requires, or whether the student can remember previous answers and use this to solve the task without substantial understanding.

Rhapsode worked differently for the four case students, and there were substantial differences in learning paths for students who have relatively high or low level of mathematical competence combined with resp. relatively high or low motivation in relation to the mathematics subject.

Rhapsode did not work optimally for student 1, who in the test and survey was in the lowest third in terms of both mathematical competence and motivation. The student lost motivation and self-confidence and was unable to get help from the support measures of the teaching aid and probably did not learn much new. Teachers must be aware that a student with relatively poor mathematical competence and motivation may need to be kept on track in relation to familiarizing himself with the teaching aid's explanations, consulting the teaching aid's feedback in case of wrong answers and investing the time necessary to answer the tasks.

Student 2 was characterized by a combination of low mathematical skills but high motivation for the subject. This was expressed by the fact that the student was very persistent despite receiving feedback from the teaching aid that he had answered many tasks incorrectly. He did not act very strategically in relation to the supporting resources in the teaching aid, but he still managed to move forward through the learning objectives until he lost heart at the end. Therefore, the learning tool worked well for this student as long as his motivation was intact.

For Student 3, who had good initial mathematics skills but low motivation for the mathematics subject, the teaching aid would be suitable as preparation for a course on a given mathematical subject area. The student has potentially become more aware of what he can do, and this will be a good starting point to build on. Even if the student scores low in terms of motivation for the mathematics subject, it seems that the student can maintain attention with Rhapsode's pace and flow.

Student 4, who scored high on both mathematical competence and motivation for the subject, the teaching aid also worked well. The module the student had chosen caused her challenges, but her thoroughness, mastery-oriented motivation and self-efficacy led her steadily and calmly towards fulfilling the learning goals.

Thus, high self-efficacy appears to be crucial for students to get a good result from working with Rhapsode. If the student gives up too easily or fails to invest effort, it does not appear that Rhapsode can create a positive and productive learning experience for the student.

The teaching tool can be used to uncover discrepancies between a student's self-efficacy and actual ability. This discrepancy can result, for example, in the student being performance-oriented in relation to answering many tasks but is not inclined to learn new things from the teaching material. In this situation, the teacher should show the student how, through the explanations and feedback of the teaching aid, he can be better equipped to solve the tasks correctly.

This study confirms the findings of previous studies as presented in the state-of-the art of this paper, that adaptive technology has not yet matured to be able to replace the teacher and make classroom level pedagogy (Du Boulay, 2019) superfluous. This is particularly relevant regarding showing students the need of using the resources in the learning material but also to broaden the scope vis-à-vis the mathematics subject. For example, students may need the teacher to present alternative ways of approaching tasks, as the teaching aid only presents one approach in explanations.

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Appendix 1. Student 1's progress through a learning module in Rhapsode and explanation and display of analytical findings

In the two rows on the left in Table 2, the 25 learning objectives are listed in column 2. The learning objectives are in the order that they have in Area9's taxonomic order. Each learning objective within a subarea is provided by us with a number for identification in the analyzes (column 1, from 1.1. to 5.1).

Table 1.
Student 1's progress through the "Division 2" module.

#	Learning objective	E1	E2	E3	E4	E5	E6	Class avg.
0	Introduction							
	Remainder							
1.1	Remember that "remainder" is what is left after dividing equally	(5)						92%
1.2	Determine the remainder in a division	(6)						100%
1.3	Perform simple division with remainder	(14)	(25)	(31)	(38)			57%
1.4	Use simple division with remainder	F (19)	50 (26)	F G (32)	(39)			58%
	Division of whole 10s, 100s and 1000s with 10s and 100s							
2.1	Divide numbers ending in 0 by 10	(2)	(8)					85%

2.2	Divide numbers ending in 00 by 10	(1)	G (7)					92%
2.3	Divide numbers ending in 000 by 10	(3)						92%
2.4	Divide numbers ending in 00 by 100	F (4)	(10)					50%
2.5	Divide numbers ending in 000 by 100	(11)	G (17)					91%
Divide numbers up to 99 by eight-digit numbers								
3.1	Divide numbers up to 99 step by step with graphical representation	F (9)	40 (15)	80 G (22)	F (28)	F G (34)	G2 (41)	77%
3.2	Divide numbers up to 99 step by step in everyday situations	44 (16)	69 G (23)	50 G3 (30)	84 G3 (35)	F G (34)	G4 (43)	73%
3.3	Divide numbers up to 99 using regrouping	F (21)	(27)	G (37)	G2 (45)	25 G3 (34)	G4 (57)	53%
3.4	Divide numbers up to 99 by 3-5 without a remainder	F (36)	(44)	G (52)	G2 (58)			41%
3.5	Divide numbers up to 99 by 6-9 without a remainder	(20)	90 G (33)	G1 (40)	G2 (47)	F (49)	G3 (55)	42%
3.6	Divide numbers up to 99 by the remainder							80%
3.7	Divide numbers up to 99 step by step with graphical representation	(53)						69%
3.8	Describe an everyday problem with a division expression							100%

Quotients of products of 10								
4.1	Use tables to divide numbers up to 1000 ending in 0 by eight-digit numbers	F (12)	(18)	G (24)				60%
4.2	Use tables to divide numbers up to 1000 that end in 0 by whole tens	(13)						35%
4.3	Divide numbers up to 1000 that end in 0 by even-digit numbers							80%
4.4	Divide numbers up to 1000 that end in 0 by whole tens							57%
4.5	Divide numbers up to 999 by a single digit number							0%
4.6	Divide numbers up to 999 using regrouping	(29)	G (42)	G2 8 (48)	F (54)			68%
4.7	Divide numbers up to 999 by 3 - 5 without a remainder	(46)	G (50)	G2 (56)				52%
4.8	Divide numbers up to 999 by 6 - 9 without a remainder							83%
Divide numbers up to 999 by the remainder								
5.1	Use division of numbers up to 999 in everyday situations	(59)						45%

In Table 1a, the rows with the 20 learning objectives that the student managed to work on in the 45 minutes he recorded are marked with colored boxes. The table also shows how many times the student has encountered a mediating sequence or task that targets each learning goal (E1, E2, etc.), as well as the student's progression and success in each activity. For each exposure, it is marked whether the student answered correctly (green), incorrectly (red), partially correct/incorrect (yellow). Numbers at partially true/false show percent correctness.

F marks that the object is an explanation, blue box with F that it is an explanation that does not test the student's understanding or skills. Numbers at partially true/false show percent correctness. F marks that the object is an explanation, and a blue box with F signals that it is only an explanation that does not test the student's understanding or skills. Numbers in parentheses show the chronology or progression between learning objectives and objects; (1) is the first learning object the student has encountered after the introduction to the module. G marks that the task is a repetition of a previous one (possibly with other values); in the case of multiple repetitions, the number of repetitions is marked by prefixing the number with G, e.g. "G2" (second repetition, i.e. third occurrence). The column on the far right shows the class's average percentage of correctness on tasks within the learning objective.

Appendix 2. Student 2's path through a module in Rhapsode.

Table 1.
Student 2's path through a module in Rhapsode.

#	Learning objective	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12	E13	Class avg.
0	Introduction														
		Commutative law													
1.1	Define the commutative law of multiplication	F (5)	(11)												71%

1.2	Apply commutative law	(6)													64%
		Multiply by 10, 100 and 1000													
2.1	Multiply a whole number by 10	F (3)	(9)	G (15)	G2 (21)	F G (29)	G3 (35)	G4 (41)							75%
2.2	Remember the rule for multiplication by 10	(42)	G (56)	F (62)	G2 (70)										46%
2.3	Multiply a whole number by 100	(1)													75%
2.4	Remember the rule for multiplying by 100	(2)	G (8)	F (14)	G2 (20)	G3 (22)	G4 (28)	G5 (36)							54%
2.5	Multiply a whole number by 1000	(4)	F (12)	G (23)	G2 (30)										70%
2.6	Remember the rule for multiplication by 1000	F (31)	G (38)	G2 (44)	F G2 (46)	G3 (52)	G4 (58)	G5 (65)	F G3 (71)	G6 (80)					55%
		Multiply numbers ending in 0													
3.1	Multiply a one-digit number and a whole 10's	(10)	G (16)												58%
3.2	Multiply two whole 10s	(27)	G (37)	G2 (47)	G3 (54)	G4 (63)	F (66)	G5 (73)	G6 (78)	G7 (84)	F G (88)	G8 (94)	G9 (102)		39%
3.3	Multiply a one-digit number and a whole 100's	F (7)	(13)	G (19)	G2 (25)	G3 (33)									50%

3.4	Multiply whole 10s and whole 100s	(17)	F (24)	G (34)											44%
3.5	Multiply whole 100s	(18)	G (26)	G1 (32)	F (39)	G2 (45)	F G (31)	G3 (57)	G4 (64)	G5 (68)	F G2 (73)	G6 (79)	G7 (85)	G8 (92)	28%
		Estimated calculation													
4.1	Identify correct rounding of two-digit factors in the context of estimates	F (99)	(105)	(109)											67%
4.2	Determine the approximate product of two two-digit numbers	(91)													67%
4.3	Determine the approximate product in everyday situations	F (7)	G (106)												67%
		Multiply one-digit and two-digit numbers													
5.1	Determine the product of a one-digit and a two-digit number up to 19 step by step using the area method	F (90)	75 (96)	75G (103)	F G (108)										91%
5.2	Multiply one-digit and two-digit numbers up to 19 with division	F (59)	F G (69)	75 (75)	75 G (81)	F G2 (86)	G2 (93)	G3 (100)							83%
5.3	Multiply one-digit and two-digit numbers up to 99 with division	(40)	75 G (49)	F (55)	(61)	G (67)	F G (74)	50 G2 (83)	G3 (89)	F G2 (95)	G3 (101)	G4 (107)	F G3 (112)	G5 (114)	38%
5.4	Multiply a one-digit and a two-digit number vertically	F (43)	F G (48)	G (53)	G2 (60)	F G2 (76)	G3 (82)	G4 (87)							50%
5.5	Calculate the product of a one-digit and a two-digit number in everyday situations	(98)	G (104)	G2 (110)											77%

		Multiply one-digit and three-digit numbers													
6.1	Multiply one-digit and three-digit numbers vertically	F (50)	(72)	G (77)											57%
6.2	Multiply one-digit and three-digit numbers	(111)													64%

Note: Learning objectives are shown in the two rows on the left of the table. Each learning objective within a sub-area is provided with a number for identification in the analyses. The table shows the number of exposures within a learning objective (E1, E2, etc.). For each exposure, it is marked whether the student answered correctly (green), incorrectly (red), partially correct/incorrect (yellow). Numbers at partially true/false show percent correctness. Blue marks that the object is an explanation, blue box with F that signifies an explanation that does not test the student's understanding or skills. Numbers in parentheses show the chronology or progression between learning objectives and objects; (1) is the first learning object the student has encountered after the introduction to the module. G marks that the task is a repetition of a previous one (possibly with other values); in the case of multiple repetitions, the number of repetitions is marked by placing the number after G, e.g. "G2" (second repetition, i.e. third occurrence). The column on the far right shows the class's average correct percentage on tasks within the learning objective.

Appendix 3. Student 3's path through a module in Rhapsode.

Table 1.
Student 3's path through a module in Rhapsode.

Ny#	Learning objective	E1	E2	E3	E4	E5	Class avg.
0	Module introduction						
	Definitions						
1.1	Identify the x-axis of a coordinate system						100%

1.2	Identify the y-axis of a coordinate system						88%
1.3	Identify the origin of a coordinate system	F					86%
1.4	Define the origin						65%
1.5	Define coordinates						52%
1.6	Describe the notation convention						60%
1.7	Identify the x-coordinate of a point given by its coordinates	F					57%
1.8	Identify the y-coordinate of a point given by its coordinates		G				65%
Identify a point given its coordinates							
2.1	Select points with the same x-coordinate						82%
2.2	Select points with the same y-coordinate						56%
2.3	Identify a point given its coordinates		G				69%
Write the coordinates of a given point							
3.1	Describe how to find the x-coordinate of a given point						90%
3.2	Describe how to find the y-coordinate of a given point						72%
3.3	Write the x-coordinate of a given a point						100%
3.4	Write the y-coordinate of a given a point						90%
3.5	Describe the meaning of the x-coordinate of a point						
3.6	Describe the meaning of the y-coordinate of a point	F					
3.7	Write the coordinates of a given point		G	G2			89%
Plot a point							
4.1	Plot a point with a given x-coordinate						100%
4.2	Plot a point with a given y-coordinate						75%

4.3	Plot a point given its coordinates	F		G	F	G2	96%
4.4	Given the point (x,y), plot the point (y,x)						47%
	Paths and shapes						
5.1	Identify a paths/shape given by points	F		75	F		81%
5.2	Write the coordinates of the corners of a path/ shape						52%
5.3	Plot the corner points of a path/shape						36%

Note: Learning objectives are shown in the two rows on the left of the table. Each learning objective within a sub-area is provided with a number for identification in the analyses. The table shows the number of exposures within a learning objective (E1, E2, etc.). For each exposure, it is marked whether the student answered correctly (green), incorrectly (red), partially correct/incorrect (yellow). Numbers at partially true/false show percent correctness. Blue marks that the object is an explanation, blue box with F that signifies an explanation that does not test the student's understanding or skills. Numbers in parentheses show the chronology or progression between learning objectives and objects; (1) is the first learning object the student has encountered after the introduction to the module. G marks that the task is a repetition of a previous one (possibly with other values); in the case of multiple repetitions, the number of repetitions is marked by placing the number after G, e.g. "G2" (second repetition, i.e. third occurrence). The column on the far right shows the class's average correct percentage on tasks within the learning objective.

Appendix 4. Student 5's path through a module in Rhapsode.

Table 1.
Student 4's path through a module in Rhapsode.

#	Learning objective	E1	E2	E3	E4	E5	E6	E7	Class avg.
0	Introduction								
	Remainder								

1.1	Remember that "remainder" is what is left after dividing equally	F (3)	(10)						92%
1.2	Determine the remainder in a division	(4)							100%
1.3	Perform simple division with remainder	(26)							57%
1.4	Use simple division with remainder in everyday situations	(28)							58%
Division of whole 10s, 100s and 1000s with 10s and 100s									
2.1	Divide numbers ending in 0 by 10								85%
2.2	Divide numbers ending in 00 by 10								92%
2.3	Divide numbers ending in 000 by 10	(1)							92%
2.4	Divide numbers ending in 00 by 100	(2)	F (9)	G (16)	G2 (22)				50%
2.5	Divide numbers ending in 000 by 100	(8)							91%
Divider tal op til 99 med etcifrede tal									
3.1	Divide numbers up to 99 step by step with graphical representation	F (5)	40 (12)	60 G (18)	F G (24)	F G2 (30)	80 G2 (39)	100 G3 (46)	77%

3.2	Divide numbers up to 99 step by step in everyd-ay situations	25 (14)	75 G (20)	75 G2 (27)	F (31)	75 G3 (37)	G4 (43)		73%
3.3	Divide numbers up to 99 using regrouping	(6)	F (15)	G (21)					53%
3.4	Divide numbers up to 99 by 3 - 5 without a remainder	(32)	G (34)	G2 (38)					41%
3.5	Divide numbers up to 99 by 6 - 9 without a remainder	(25)	G (45)						42%
3.6	Divide numbers up to 99 by the remainder	F (44)							80%
3.7	Use division of numbers up to 99 in everyday situations	F (33)	F (40)						69%
3.8	Describe an everyday problem with a division expression	(51)							100%
	Quotients of products of 10								
4.1	Use tables to divide numbers up to 1000 ending in 0 by eight-digit numbers	F (7)	(13)	G (19)	(52)	G (54)			60%
4.2	Use tables to divide numbers up to 1000 that end in 0 by whole tens	(17)	G (23)	F (29)	(35)	(42)	G (49)		35%
4.3	Divide numbers up to 1000 that end in 0 by even-digit numbers	F (48)	F G (55)						80%
4.4	Divide numbers up to 1000 that end in 0 by whole tens	F (11)	(50)						57%

4.5	Divide numbers up to 999 by a single digit number								0%
4.6	Divide numbers up to 999 using regrouping	(36)							68%
4.7	Divide numbers up to 999 by 3 - 5 without a remainder	(34)							52%
4.8	Divide numbers up to 999 by 6 - 9 without a remainder								83%
	Divide numbers up to 999 by the remainder								
5.1	Use division of numbers up to 999 in everyday situations	F (41)	(47)	70 (53)	G (56)				45%

Note: Learning objectives are shown in the two rows on the left of the table. Each learning objective within a sub-area is provided with a number for identification in the analyses. The table shows the number of exposures within a learning objective (E1, E2, etc.). For each exposure, it is marked whether the student answered correctly (green), incorrectly (red), partially correct/incorrect (yellow). Numbers at partially true/false show percent correctness. Blue marks that the object is an explanation, blue box with F that signifies an explanation that does not test the student's understanding or skills. Numbers in parentheses show the chronology or progression between learning objectives and objects; (1) is the first learning object the student has encountered after the introduction to the module. G marks that the task is a repetition of a previous one (possibly with other values); in the case of multiple repetitions, the number of repetitions is marked by placing the number after G, e.g. "G2" (second repetition, i.e. third occurrence). The column on the far right shows the class's average correct percentage on tasks within the learning objective.